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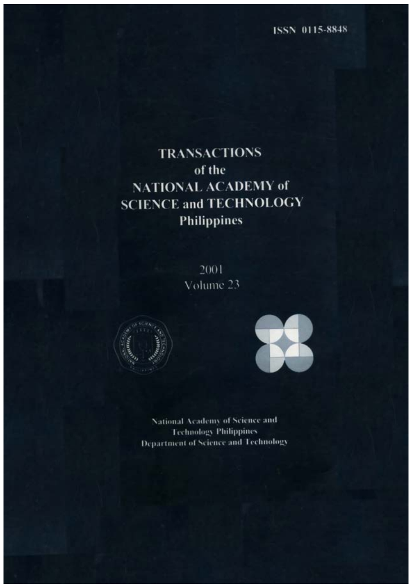

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CHAOS IN THE ACTIVATED SLUDGE PROCESS

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ABSTRACT

The activated sludge process is used to remove biodegradable organic impurities in wastewater using bacteria and other microorganism to decompose the organic impurities into water, carbon dioxide, and additional microbial cells. One of the main problems in the operation of the activated sludge process is the occurrence of sudden and often unexplained process instability or upsets. A number of hypothesis have been developed to explain the nature and cause of process instability. The process kinetics involved a number of non-linear and linear differential equations to define the prey-predator relationship among various microbial species, the dominance and predominance of specific species to the presence of poisons, biochemical catalysts, and type of subtrate. The kinetic models of the activated sludge process is very complex.

Using the basic kinetic model for the completely mixed activated sludge process, the chaotic behavior of the system was studied. The study also identify the process and design variables creating chaos and estimate the ranges of those design and operating variables within which chaos takes place. The study confirmed that the activated sludge process is unstable when operated under (i) a very short detention time of less than 2 hours; (ii) very stringent effluent quality, i.e. less than 2 mg/l, and (iii) very high influent COD concentration with detention time set at 4 hours or less. Increasing the detention time increases the system stability.

INTRODUCTION

The activated sludge process is used to remove biodegradable organic impurities in wastewater using bacteria and other microorganisms to decompose the organic impurities into water, carbon dioxide and additional bacterial cells in the presence of excess quantities of oxygen. The control and process development of the activated sludge were developed empirically and mathematical models were only developed in the late 1960s when computers with high speed and large memory became available. Exact mathematical solutions of the activated sludge process system equations are possible only for a limited number of operating conditions due to a number of non-linear and linear differential equations which must be solved simultaneously.

One of the main problems in the operation of the activated sludge process is the occurrence of sudden and unexplained process instability or upsets. Sudden change in temperature, influent flow rate, influent organic matter concentration, trace concentration of poisons and biochemical catalysts in the wastewater, and prey-predator relationship, as well as changes in the growth and metabolic patterns of the microbial populations as they interact with each other are some of the hypotheses suggested to explain process upset n the activated sludge. With the large and fast computing capability of new digital computers, the activated sludge models evolved into more complex ones a more variables, linear and non-linear differential equations are added to define the intricacies of the system.

OBJECTIVES AND SCOPE OF THE STUDY

The main objective of the study is to examine and define the regions of chaos using the basic kinetic equations and a system of differential equations modelling the behavior of a completely mixed variations of the activated sludge process. The secondary objectives of the study are (i) to identify the process of design variables creating chaotic behavior in the activated sludge process and (ii) to estimate the ranges of those design and operating variables within which chaos takes place.

The scope of the study includes (i) the development of a computer model for the completely mixed activated sludge process, (ii) testing the model under various ranges of design and operating variable using the kinetic constants and typical characteristics of domestic sewage, and (iii) identification of the regions where chaotic behavior takes place.

MODEL DEVELOPMENT

A. Process Description

The activated sludge process consists of an aeration tank and a secondary clarifier. A grit removal and primary clarifier is often used before the aeration tank to remove coarse solids and grit scoured or accidentally added to the sewage during transport in the sewer system. Grit and coarse particles have tendencies to settle in the aeration tank and reduce the effective aeration tank volume. In the analysis of the biochemical reaction in the activated sludge process, the grit and primary sedimentation tanks is normally excluded. The biochemical reaction primarily takes place in the aeration tank where a mixed population of microorganism converts the organic pollutant into water, carbon dioxide and additional microbial cells in the presence of excess quantities of oxygen. In the aeration tank the microbial population composition is constantly changing because of the prey-predator relationship, varying adaptation capacities of the organism to the changing environmental conditions such as quantity and type of food and nutrient available, temperature, sunlight, micro-impurities in the water such as hydrogen ion, iron and biochemical secretions from competing and symbiotic microbial populations (Unz and Williams, 1988). The microbial population also undergoes endogenous respiration decaying into inert microbial matter and releasing the stored microbial nutrient into solution

The active and inert microbial organisms or microbial masses are separated from the treated wastewater in the secondary clarifier. The microbial mass settles at the bottom of the clarifier to a consistency similar to septic tank sludge. It is then recirculated to the aeration tank for the active microbial mass to initiate another cycle of destruction of the organic pollutant and production of additional active microbial mass. A high concentration of active microbial mass, often equivalent to the microbial mass produced in 30 to 50 days, is maintained in the aeration tank. Such high concentration of active microbial mass in the aeration tank reduces the time required to destroy the organic pollutant in the activated sludge process when compared to other wastewater treatment processes such as the aerated or facultative pond system. Excess active microbial mass in the secondary clarifier is periodically removed to maintain the sludge level at 1 m below the treated wastewater level in order to minimize the suspended solids in the effluent. The schematic diagram of the process is shown in Fig. 1.

A number of variations of the activated sludge process have been developed depending on (i) mixing and flow pattern in the aeration tank, (ii) average aeration time or detention time, (iii) mode of introduction of the raw wastewater in the aeration tank, and (iv) the mode of oxygen aeration. The mixing pattern in the aeration tank could be classified as well-mixed, commonly called completely mixed

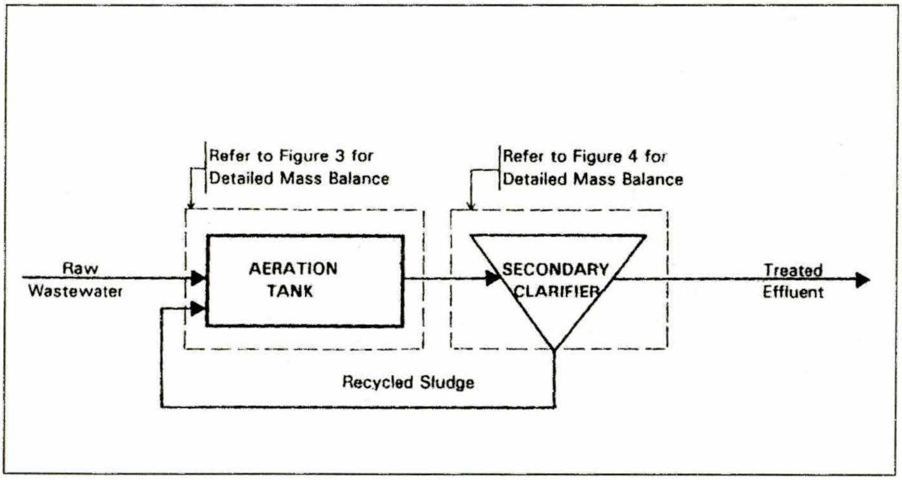

Figure 1. Schematic Diagram of the Activated Sludge Process

activated sludge process or as plug flow commonly called conventional activated sludge process. To maintain flow patterns close to plug flow, baffles are added dividing the aeration tank to a series of completely mixed tank and to minimize horizontal dispersion of the reactants in the aeration tank. The sections behave like a series of completely mixed activated sludge. A high rate activated sludge is operated at reaction time of three hours or less with high recirculation rate and concentration of active microbial mass. While this variation of the activated sludge process has the advantageous minimum aeration tank volume requirement it is, however, subject to frequent process upset or instability and is seldom used except as a pretreatment process. The usual aeration time is from four to eight hours for most of the variations of the activated sludge process except for the extended aeration process which is operated at 9 to 30 hours aeration time. The extended aeration process is more stable although the operating cost is higher as a high proportion of the active microbial mass is converted into inert matter requiring higher quantities of oxygen per unit weight of organic pollutant stabilized. In a plug flow activated sludge process, the oxygen demand is higher at the inlet where the organic pollutant concentration is highest gradually dropping towards the outlet. The step aeration system aims to match the oxygen supply with the oxygen consumption rate thereby minimizing the energy required to maintain aerobic conditions in the aeration tank. The step feed system maintains the aeration rate constant throughout the length of the aeration system and spread the introductions of the raw wastewater more evenly. The schematic diagrams of the various variations of the activated sludge process are shown in Fig. 2.

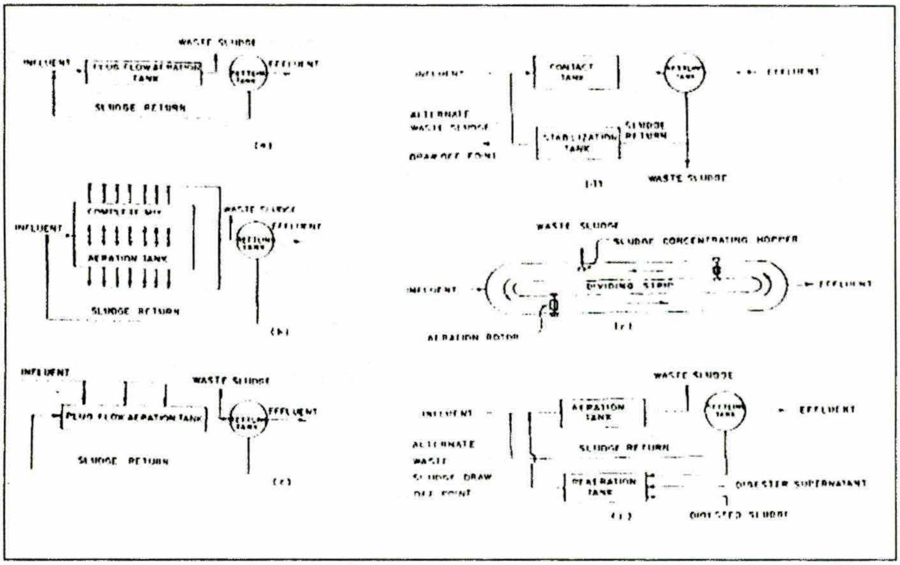

Figure 2. Variations of the Activated Sludge Process

Plug flow in the aeration tank is considered more efficient than the completely mixed system although the difference in performance is quite small. In an autocatalytic reaction Lievenspiel (1962) has shown that a completely mixed fermentation reactor which is similar to a completely mixed activated sludge process is more efficient than a plug flow fermentation reactor when substrate is in reactor when substrate is the rate limit factor. The most efficient system is a combination of the two systems. The high recirculation rate in the activated sludge system which builds up the active microbial mass population in the aeration tank induces mixing conditions approaching the completely mixed reactor (aeration tank) even if the system is designed as a plug flow reactor. Due to the simplicity in the design and construction of the completely mixed activated sludge system over other variations of the activated sludge process described above, the completely mixed activated sludge system is more widely used today than any other variations. The extended aeration system is a completely mixed activated sludge system with long detention time of 20 hours to 3 days and sludge recirculation rate of 100 to 300 percent. The aerated lagoon is similar to the completely mixed activated sludge system if the recirculation rate is zero and the detention time is longer than 3 days. Even the activated sludge variations with plug flow patterns could be modelled as a series of completely mixed activated sludge system operating in series. For this reason the complete mixed activated sludge process is used in this study.

B. Basic Kinetics and Process Equations

The microbial mass production rate, dX/dt, was established by Monod (1949) and is commonly called Monod's equation. Monod's equation is:

$$\frac{1}{X}\frac{dX}{dt} = \mu_{max}\frac{S}{K_m + f(x, S)} \qquad \text{Eq. (1)}$$

where X is the microbial mass, mg/l μ_{max} is the specific growth rate hr^{-1}, dimensionless; K_m is the Monod's constant, mg/l; f(X,S) is a variable dependent on X; and S is the substrate concentration, mg/l. In the original equation derived by Monod f(X,S) is equal to S but for wastewater a number of variations have been proposed which gives better fit to the experimental results. Gaudy and Ramanathan (1971) proposed that f(X,S) is equal to the initial microbial mass concentration, X_o, IAWPRC (1984 and 1994) proposed the initial substrate concentration, S_o; while in most experimental studies and literature on operating activated sludge process such as Eckenfelder (1962), Schroeder (1975) and Metcalf and Eddy (1974) f(x) is ignored. For purposes of the model used in the study, the microbial growth rate equation is simplified to:

$$\frac{dX}{dt} = K_2 + XS \qquad \text{Eq. (2)}$$

where K_2 is a constant, l/mg-hr and other terms are as previously defined.

In biochemistry a substrate is defined as a mixture of essential nutrients such as nitrogen, phosphorus, trace quantities of iron, calcium, manganese, magnesium and base organic material primarily carbohydrate such as starch or sugar. In wastewater treatment literature, a substrate is often defined as the organic pollutant expressed as chemical oxygen demand (COD) or biochemical oxygen demand (BOD). The organic pollutant in wastewater is composed of a wide range of organic substances whose common effect is the depletion of the dissolved oxygen in the receiving waterbody when they decompose. As environmental engineers are concerned not with the decomposition pathways and by products of specific organic compounds but with the effects of the decomposition of materials on the receiving waterbody, the COD and BOD tests were developed as empirical estimates of the concentration of organic potassium dichromate required to completely oxidize the organic impurities in one liter of wastewater. The COD test is the milligram of dissolved oxygen consumed during five days of decomposition of organic impurities at 20°C by an acclimatized colony of microorganism. As the COD test is faster and more convenient it is more widely used than the BOD test today and is used in this study. The BOD/COD ratio is often used as an indicator of the wastewater treatability as it is an estimate of the ratio of organic impurities biologically decomposed in five days to the total quantity of organic impurities destroyed by a strong oxidizing agent. Wastewater with BOD/COD ratio above 0.3 are normally treated in a biological wastewater treatment plant such as the activated sludge and physico-chemical process such as coagulation and flocculation if the BOD/COD ratio is lower than 0.3. The rate of new microbial mass production is proportional to the quantity of substrate consumed. The proportionality constant is commonly called the yield coefficient, Y which is equal to the microbial mass produced per unit of substrate consumed. From equation 2, the rate of COD reduction is equal to:

$$\frac{dS}{dt} = \frac{-K_2 + XS}{Y} = -K_1 + XS \qquad \text{Eq. (3)}$$

where K_1 is the rate of substrate stabilization, l/mg-hr^{-1} and the other terms are as previously defined.

A portion of the active microbial mass dies out in the aeration tank. The die out rate is proportional to concentration of active microbial mass, or

$$\frac{dX}{dt} = -K_3 X \qquad \text{Eq. (4)}$$

where K_3, is a constant, hr^{-1} and the other terms are as previously defined. The production rate of inert microbial mass, X_i in mg/l, is proportional to the concentration of active microbial mass dying, or

$$\frac{dX_i}{dt} = K_4 X \qquad \text{Eq (5)}$$

where K_4 is a contant, hr^{-1} and the other terms are as previously defined. The inert mass produced is approximately 10 to 20 per cent of the dead microbial mass since a large portion of the dying microbial mass is converted to carbon dioxide or released back into the water to recycle the nutrient for consumption by the active microbial mass.

C. Model Development

The diagram of the microbial and substrate mass inflow and outflow streams in the aeration tank is shown in Fig. 3. The substrate input streams are (i) the untreated substrate which is equal to the product of the influent flow rate Q and the untreated wastewater COD, S_o, and (ii) the recycled substrate from the secondary clarifier which is equal to the product of the recirculation rate, Q_r and the effluent COD, S_e. The substrate output streams are (i) the decomposed substrate which is equal to the product of the aeration tank volume, V, and the substrate decomposition rate as given in Equation 3; and (ii) the residual substrate after treatment which is equal to the product of the sum of the influent flow and recycled flow $(Q + Q_r)$ and the effluent substrate concentration, S_e, from the aeration tank. The substrate

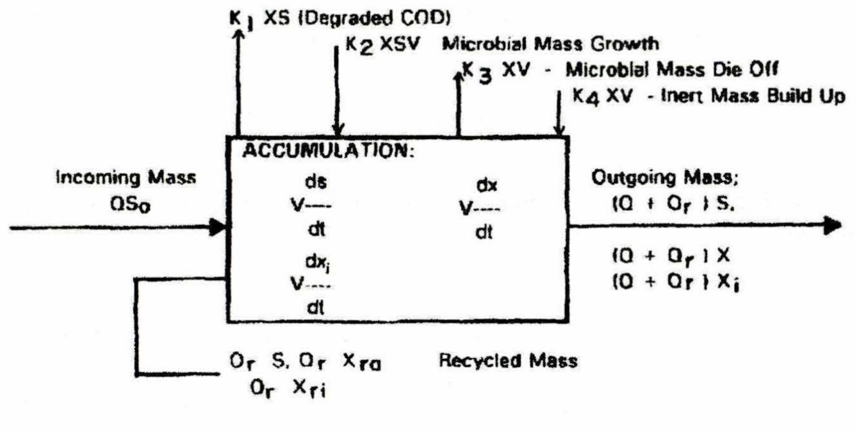

Figure 3. Mass Balance Diagram in the Aeration Tank

accumulation in the aeration tank is equal to the product of the aeration tank volume, V and the change in the substrate concentration, dS/d/t. The substrate accumulation is equal to the sum of the inputs less by the outputs. Hence.

$$V\frac{dS}{dt} = QS_o + Q_rS - K_1XSV - (Q_r + Q)\,S \qquad \text{Eq (6)}$$

or rearranging to collect similar terms, yield,

$$\frac{dS}{dt} = \frac{Q}{V}\,(S_o - S) - K_1XS \qquad \text{Eq (7)}$$

where the terms are as previously defined.

From Fig. 3, the active microbial mass inputs in the aeration tank consist of (i) the microbial mass produced during the decomposition of the substrate which is equal to the product of the aeration tank volume, V, and the microbial mass growth rate given in equation 2 and (ii) the active microbial mass recycled from the clarifier which is equal to the product of the recirculation rate Q_r and the concentration of the microbial mass in the clarifier, X_{ra}. The microbial mass outputs are (i) the active microbial mass dying rate which is equal to the product of the aeration tank volume, V and the die out rate as given in equation 4, and (ii) the microbial outflow which is equal to the product of the outflow rate (Q_r + Q) and the microbial concentration in the aeration tank. The rate of active microbial mass accumulation in the aeration tank is equal to the product of the aeration tank volume and the changes in the active microbial mass concentration in the aeration tank, dX/dt. The accumulation rate of the active microbial mass in the aeration tank is also equal to the sum of the inputs less the sum of the outputs, or.

$$V\frac{dX}{dt} = Q_rX_{ra} - (Q + Q_r)\,X + K_2XSV - K_3XV) \qquad \text{Eq (8)}$$

Equation 8 could be the rearranged and simplified by collecting similar terms which yields:

$$\frac{dX}{dt} = \frac{Q}{V}\left\{\frac{Q_r}{Q}(X_{ra} - X) - X\right) + K_2XS - K_3X) \qquad \text{Eq (9)}$$

From Fig. 3 the inert microbial mass input in the aeration tank consists of (i) the inert microbial mass recycled which is equal to the product of the recirculation rate Q_r and the inert microbial mass concentration in the recycled sludge, X_{ri} and (ii) the inert microbial mass produced by the dying active microbial mass which is equal to the product of the aeration tank volume V and Equation 5. The inert microbial mass output is equal to the product of the outflow rate (Q + Q) and the inert microbial mass concentration in the aeration tank, X_i. The accumulation rate of the inert microbial mass in the aeration tank is equal to the product of the aeration tank volume, V and the changes in the inert microbial mass concentration in the aeration tank, dX/dt. The accumulation rate of inert microbial mass in the aeration tank is equal to the sum of the inputs less the sum of the outputs. Hence,

$$V\frac{dX_i}{dt} = Q_rX_{ri} - (Q + Q_r)\,X_i + K_4XS - K_3XV) \qquad \text{Eq (10)}$$

Equation 10 could be simplified by collecting similar terms or

$$\frac{dX_i}{dt} = \frac{Q}{V}\{\frac{Q_r}{Q}(X_{ri} - X_i) - X_i\} + K_4X \qquad \text{Eq (11)}$$

Fig. 4 is a schematic diagram of inputs and outputs streams in the secondary clarifier. Unlike in the aeration tank where turbulent conditions are preferred in order to maximize the contact between the reacting substances, the secondary clarifier is maintained at quiescent conditions so that the particles will settle and compact at the bottom of the tank with minimum uplift current. The microbial floc or particles from the aeration tank come in various sizes. When those particles settle in the secondary clarifier they have different settling velocities. The larger particles collide with smaller and slower moving particles forming larger floc. Those phenomenon is commonly referred to as flocculent settling. As a result a distinct clear water/sludge interface develops in the secondary clarifier. If the sludge stays for more than three hours in the secondary clarifier, the floc become

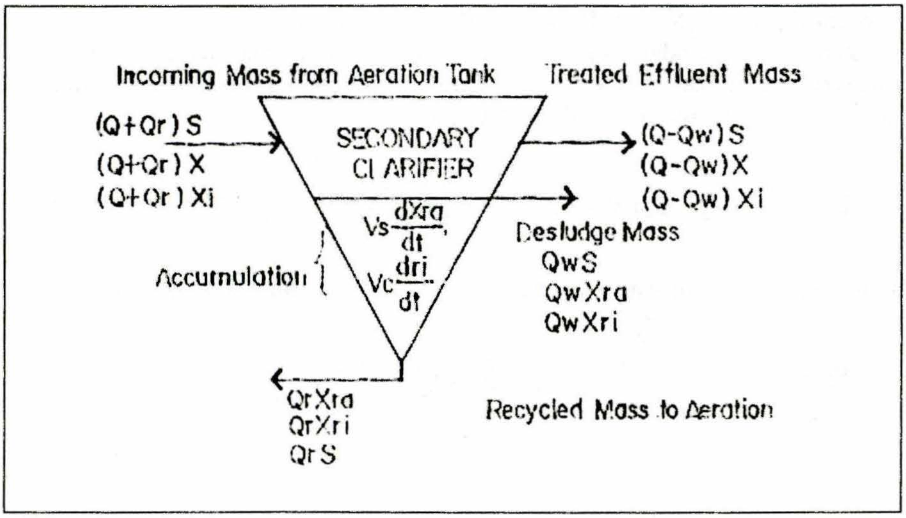

Figure 4. Mass Balance Diagram in the Secondary Clarifier

anaerobic and starts to float as the floc density is reduced by the methane, carbon dioxide, nitrogen, hydrogen sulfide and other gases generated during anaerobic fermentation of the sludge. On the other hand very high recirculation rate prevents the sludge from compacting properly resulting in dilute sludge concentration and reduced operating flexibility in controlling the active microbial mass concentration in the aeration tank.

Excess sludge in the secondary clarifier is removed intermittently. A sonar device measures the thickness of the sludge blanket and the sludge concentration. Once the sludge blanket reaches more than 30 per cent of the secondary clarifier depth, or the sludge concentration exceeds 2 per cent, the desludging pump automatically starts.

From Fig. 4, the active microbial mass input to the secondary clarifier is the outflow from the aeration tank which is equal to the product of the sum of the untreated wastewater flow rate and the recirculation rate ($Q + Q_r$) and the active microbial sludge concentration in the aeration tank X. The active microbial outputs are (i) the active microbial sludge carried out with the treated water which is equal to the product of the treated water flow rate ($Q\text{-}Q_w$) and the concentration of active microbial mass in the effluent (X_{ea}); (ii) the recycled active microbial mass which is equal to the product of the recirculation rate (Q_r) and the settled active microbial mass concentration (X_{ra}); (iii) the active microbial mass desludged which is equal to the product of the desludging rate (Qw) and the settled active microbial mass concentration (X_{ra}); and (iv) the active microbial mass that dies out which is equal to the product of the active microbial mass concentration, X_e, the clarifier tank volume, V, and the die off constant K_3. The active microbial mass concentration accumulation in the secondary clarifier is equal to the rate of change of the product of the settled sludge volume (V_s) and the active microbial sludge concentration, $d(V_s X_{ra})/dt$. If the settled sludge volume is maintained and almost constant by the desludging operation, then the active microbial sludge accumulation rate could be expressed as $V_s dX_{ra}/dt$. The active microbial mass accumulation is equal to input less the sum of the outputs, or

$$Vs\frac{dX_{ra}}{dt} = (Q + Q_r)X - (Q - Q_w)X_{ea} - Q_r X_{ra} - Q_w X_{ra} - K_3 X_{ra} V_s) \quad \text{Eq (12)}$$

Equation 12 could be simplified by collecting similar terms which yields

$$\frac{dX_{ra}}{dt} = \frac{Q}{V_s}\{X(1 + \frac{Q_r}{Q}) - (1 - \frac{Q_w}{Q})X_{ea} - (\frac{Q_r}{Q} + \frac{Q_w}{Q})X_{ra}\} - K_3 X_{ra} \quad \text{Eq (13)}$$

The input, output and accumulation of the inert microbial mass in the secondary clarifier is similar to those of the active microbial mass except that X_{ri} is used instead of X_{ra}, X_{ei} instead of X_{ea} and X_i instead of X and the appropriate kinetic constants. The mass balance of the inert microbial mass yields:

$$Vs\frac{dX_i}{dt} = (Q + Q_r)\,X_i - (Q - Q_w)\,X_{ei} - (Q_r - Q_w)\,X_{ri} + K_4 X_{ri} V_s) \quad \text{Eq (14)}$$

Equation 14 could be simplified by collecting similar terms to:

$$\frac{dX_{ri}}{dt} = \frac{Q}{V_s}\{(1 + \frac{Q_r}{Q})\,X_i - (1 - \frac{Q_w}{Q})\,X_{ei} - (\frac{Q_r}{Q} + \frac{Q_w}{Q})\,X_{ri}\} + K_4 X_{ra} \quad \text{Eq (15)}$$

D. Operating Procedures

The effluent COD should be less than or equal to the effluent standard set by the regulatory agency. In most countries the effluent COD is set at 40 mg/l corresponding to a BOD concentration of 30 mg/l. The operator increases the sludge recirculation rate in order to increase the active microbial mass concentration in the aeration tank and subsequently the COD removal rate as defined by Equation 7. The target active microbial mass concentration in the aeration tank, X_t is derived from Equation 7 by substituting the effluent standard, S_e for S which leads to:

$$X_t = \frac{Q}{K_1 S_e V}\,(S_o - S_e) - \frac{1}{K_1 S_e}\,\frac{dS}{dt} \quad \text{Eq (16)}$$

The recirculation rate is then calculated from Equation 9 by substituting Xt for X which results to:

$$\frac{Q_r}{Q} = \frac{\mathrm{V}}{Q(X_{ra} - X_t}\,\{\frac{dX}{dt} + \frac{QX_t}{V} - K_2 S_e X_t + K_3 X_t\} \quad \text{Eq (17)}$$

Desludging is carried out intermittently when the sludge in the clarifier exceeds the volume Vs and if the sum of the inert and active microbial mass exceeds as specified limit such that the sludge settling is hindered or the sludge becomes septic due to long residence time in the clarifier. For purposes of the model sludge desludging will take place when the sum of the inert and active microbial mass concentration in the clarifier exceeds 20,000 mg/l (2 per cent) and the desludging is stopped when the concentration goes below 18,00 mg/l. The desludging rate is taken at one per cent of the average untreated wastewater flow rate.

E. Model Constants and Assumptions

The values of the kinetic constants are taken from textbooks and literature as shown in Table 1. The initial conditions of the model are selected for the system to attain steady state conditions in order to reduce the time required for the model to remove the transient effects. The initial conditions are also given in Table 1.

Table 1. Summary of Constants

Parameters	Range of Values[1]	Values Used in the Model
K1	0.013 to 0.036	0.03
Y	0.4 to 0.67	0.5
K2	0.008 to 0.144	0.015
K3	0.25 to 0.60	0.40
K4	0.05 to 0.15	0.10

F. The Model

The activated sludge model consists of the mass balance equations for (i) the substrate (COD), (ii) the active microbial mass, (iii) inert microbial mass, (iv) active microbial mass in the recycled solids, (vi) recirculation rate required to attain the effluent standard, and (vii) desludging criteria. The differential equations for the substrate, active microbial mass, inert microbial mass, active mass recycled and inert mass recycled were solve using the Runge-Kutta method for systems of equations with five variables. The required recirculation and desludging rates are calculated algebraically from Equations 16 and 17. Discussion on the Runge-Kutta method used for the model is given in most textbooks on numerical analysis such as Froberg (1969) and McCalla (1967). The computer program is written in Trubasic language[2] to make the model compatible with programs developed by Baker and Gollub (1990) for higher methods of analyzing chaotic behavior of systems of differential equations with one or more non-linear equations and simulation of chemical processes (Hanna and Sandall, 1995). The computer program is given in Appendix 2.

G. Limitations of the Model

The model developed does not consider the fluctuations in the untreated wastewater flow rate and substrate concentration. The fluctuations in the untreated wastewater flow rate and substrate concentration are the superposition of the cyclic patterns of various frequencies, i.e. hourly, eight hour shift of the factories,

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[1] Values were calcuated from data from Gujer and Henze (1991), Sollfrank and Gujer (1991), Nakazawa and Tanaka (1991), Mckinney (1962 and 1974) and McCarty and Brodersen (1960).

[2] Trubasic is a registered trademark of True Basic Inc., of West Lebanon, NH., USA.

daily, weekly, monthly, seasonal, annual and longer frequencies. Future models could express the influent flow rate as:

$$Q\,(t) = Q \sum^{n=0} a_n \cos\,(2\pi\alpha_n t) \geq 0 \qquad \text{Eq (18)}$$

and the substrate as

$$Q\,(t) = S(b_o + \sum^{n=1} b_n \sin\,(2\pi\beta_n t) \geq 0 \qquad \text{Eq (19)}$$

where Q_m is the mean influent flow rate, cm/day, S_m is the mean substrate concentration, Q_n are the frequencies of the influent flow rate, β_n are the frequencies of the substrate concentration, a_n is the amplitude of the wastewater flow rate, b_o and bn are the amplitudes of the wastewater COD concentration. The fluctuations in the influent flow rate and substrate concentration increase the non-linearity of the equations in the activated sludge process model and would be of interest in more detailed analysis of the chaotic behavior of the process.

The microbial population in the activated sludge is a mixture of wide range of a species of bacteria, protozoa, fungi, and algae existing in symbiotic and/or prey-predator relationship. Protozoa prey on the bacteria specially the unicellular non-flocculent bacteria and are primarily responsible for the clarity of the effluent. Nitrifying bacteria produce nitrites and nitrates once the substrate levels become low (Simpkins, 1988). The growth of predators conforms to Equation 3 and the demise of the prey to Equation 4. In more complex models of the activated sludge process, it is not unusual to find seven additional non-linear equations defining the growth and decay of prey and predators (Andrews, 1974, IAWPRC, 1987 and IWAPRC, 1994). While the more complex models are useful to explain some aspects of the activated sludge process, the existence of chaotic behavior is easier to identify using the simpler model as previously described. This study is an exploratory work on the chaotic behavior of the activated sludge process and it is essential that the simplest model is used.

CHAOS THEORY

While Chaos Theory in its present form is a new branch of mathematics, the foundation of the theory could be traced to the studies of Volterra and Poincare on population dynamics and prey predator relationship (Davis, 1962). With a fixed supply of solar energy, a continuous fixed supply of food is available. Theoretically the food consumer population will reach a constant level in harmony with the food supply (Kingsland, 1985). However, the food consumer population growth rate gains momentum and its number increases to a level above that which could be sustained by the food supply. Similarly, the available food supply drops below the normal level due to over consumption. As a result the food consumer population decreases to level lower than the expected equilibrium population and the available food supply starts to increase. The consumer population fluctuates in sinosoidal pattern. The available food supply follows the same pattern lagging by approximately 90 degrees. The interrelationships between grass-sheep or the deer-mountain lion population described in ecology textbook are examples of this phenomenon. This leads Poincure to develop the concept of a limit circle for a system of non-linear differential equations. The set of differential equations of the food consumer and the available food follow a circular pattern with the locus of one equation chasing the other within the limit circle. As the food consumer life cycle decreases, the frequency of the sinusoidal patterns decreases and abrupt changes in the population food supply level will take place. The disruption could be temporary with the pattern setting back to the original sinusoidal pattern after the "shockload" or cause of the abrupt change is removed. In some instances the disruption is permanent (May, 1974 and 1976).

Chaos theory in its present form is credited to the studies Lorentz made on weather prediction. After World War II, the US Weather Bureau started an ambitious weather prediction study. As the mathematical models were found inadequate, more complex models were developed by adding variables and systems of differential equations which were often non-linear. The same trend is currently popular in activated sludge modelling. Lorentz proved that by adding more variables and non-linear differential equations into the model, the results were more erratic and further from reality. Chaos could take place in a system consisting of one non-linear differential equation such as the equation of a pendulum as described by Baker and Gollub (1991). Chaos is an inherent characteristic of a system of three or more differential equations with at least one of the differential equation containing a non-linear term[1] (Slotine and Li, 1991 and Ott, 1993). The mode of the activated sludge process described in the previous section is a system of five differential equations with two of the differential equations containing a non-linear term. The operating procedure of adjusting the recirculation rate is another non-linear factor. The change in the recirculation rate is calculated from the substrate and microbial mass balance equations which are non-linear differential equations. In fact the chaotic behavior in

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[1] Chaos and random events are often confused in the few literatures on the application of chaos theory in environmental engineering. For example, a recent paper by Angelek and Minkara (1994) discussed chaos in wastewater flow which was erroneos as the wastewater flow was not defined by a specific equation. Random events are results of processes which are not defined by a mathematical equation. For example the result from the throw of a die is a random event. The die could give any value between one and six occurring without specific logical sequence. Chaos occur when a variable or variables are defined by specific sets of equations and for a specific ranges of the indepenent variable, commonly known as the chaotic ranges, the dependent variable behaves unpredictably. Outside the chaotic range, the same equation could be deterministic that is for a particular value of the independent variable there exist a corresponding and specific value of the dependent variable. The methods of quantifying the intensities of randomness and chaos are the same which leads to the confusion.

ecology studied by May (1974 and 1976) was similar to the growth rate and substrate consumption patterns defined by Monod's equation.

Chaos theory is not an accepted branch of mathematics as standard mathematical proofs and procedure could not be applied (Gleick, 1987). The regions and occurrence of chaotic events are determined by numerical methods or mechanical procedures using computers. At present a number of techniques in non-linear systems theory such as the Poincare section of the phase plane are used. As an initial investigation on the chaotic nature of the activated sludge process, this study focuses on the initial mapping and identification of the chaotic regions of the process.

RESULTS AND DISCUSSION

A. Effects of the Aeration Time

Optimization of the activated sludge design is focused on the reduction of the aeration tank, the oxygen consumption and the sludge production rate. The aeration tank is the biggest civil work component and a major factor in determining the land area required and location of the facilities. To reduce the aeration tank volume, process development focused on the reduction of the reaction time through an increase in the active microbial mass concentration in the aeration tank which is achieved by either increasing the sludge recirculation rate or increasing the recycled sludge concentration by adding a thickener after the sedimentation tank. The increase in the active microbial mass concentration in the aeration tank is not proportional to the recirculation rate or the concentration of recycled microbial mass as the microbial mass build up in the aeration tank is accompanied by an increase of inert microbial mass. In terms of the operation of the process, the major cost item is the cost of power to supply the oxygen. Increasing the active microbial mass in the aeration tank results in longer aeration time and increased oxygen consumption by the dying microbial cells' endogenous respiration. However, additional power cost to supply the oxygen is compensated by the lower quantity and improved quality of the sludge in terms of odor, dewatering characteristics, chemical consumption and storage requirements. For this reason the development of more efficient oxygen transfer equipment is a preferred option to the reduction of the oxygen consumption to reduce the operating cost.

The computer model was run with an aeration time of 5 days to test the behavior of the system at long detention time although sludge plants are not actually designed to operate at this condition. The model output is shown in Fig. 5. While the substrate input is constant, the food consumer cycle is evident. The effluent substrate (residual food) concentrating fluctuates in a sinusoidal pattern. The model operated with the influent COD and effluent COD standard maintained at 250 mg/l and 30 mg/l pattern with shorter frequency but of a regular pattern as shown in Fig. 6. There are more abrupt changes from maximum to minimum values of the effluent substrate concentration at 0.25 days detention time compared to the output at 5 days aeration

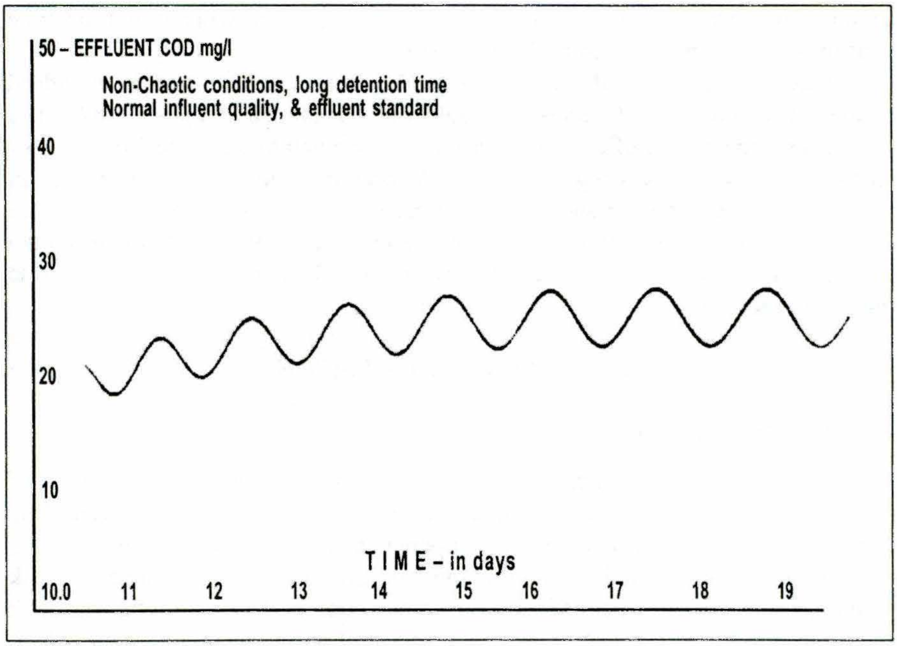

Figure 5. Effluent COD concentration Fluctuation at Long Detention Time

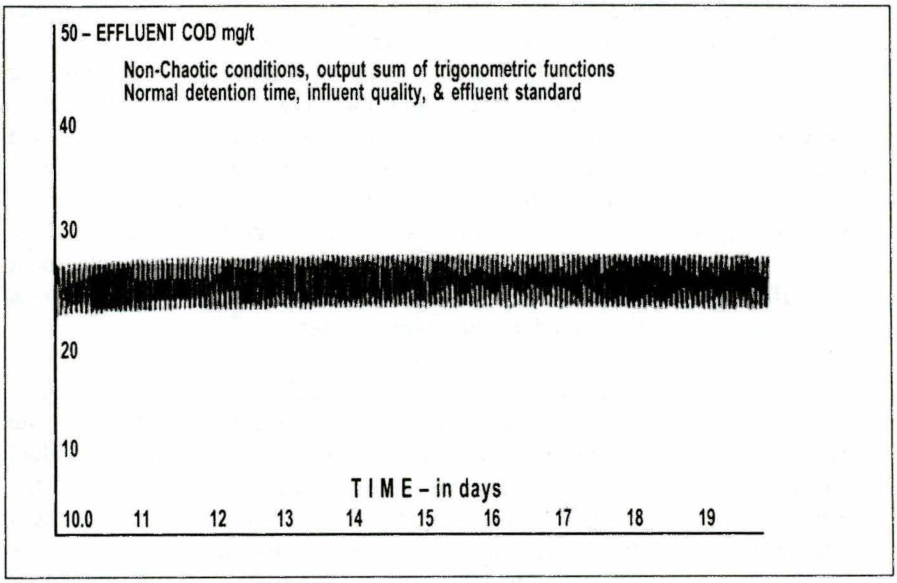

Figure 6. Effluent COD concentration Fluctuation at Normal Detention Time

time. The output is typical of the effluent from extended aeration or completely mixed activated sludge. The fluctuations in the effluent quality are often undetected since occurs in a narrow band with short frequencies which are readily averaged even in samplers operating with short sampling time.

The detention time was reduced to 0.1 of a day while the influent and required effluent concentrations were retained at 250 and 30 mg/l respectively. The output effluent concentration varied drastically without any pattern as shown in Fig. 7. The process is operating under chaotic conditions. The high rate activated sludge is designed and operated at detention times of 2 to 3 hours (0.08 to 0.125 day). The high rate activated sludges in very sensitive to minor changes in the operating conditions and exhibits frequent process upsets. For this reason the high rate activated sludge is used as a pre-treatment system. The effluent from the high rate activated sludge is treated further to remove the fluctuations or variations of the effluent quality.

B. Effects of Effluent Substrate Target

The model was tested with influent COD of 250 mg/l and detention time of 0.25 day but a target effluent COD concentration of 2 mg/l. The effluent COD concentration is shown in Fig. 8. The model is operating under chaotic conditions. The main cause for the chaotic behavior is the large fluctuations in the activated sludge recirculation rate required to build up the microbial in the aeration tank.

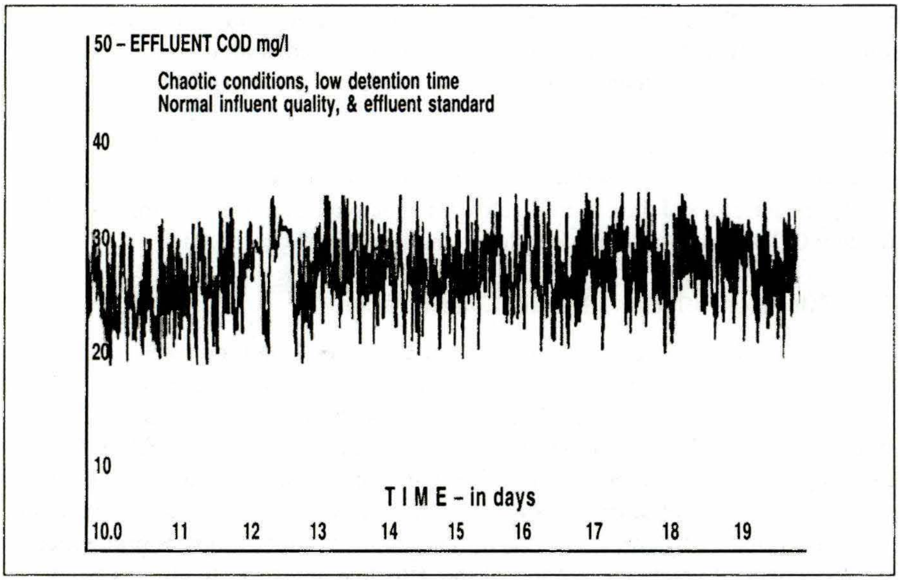

Figure 7. Effluent COD concentration Fluctuation at Short detention Time

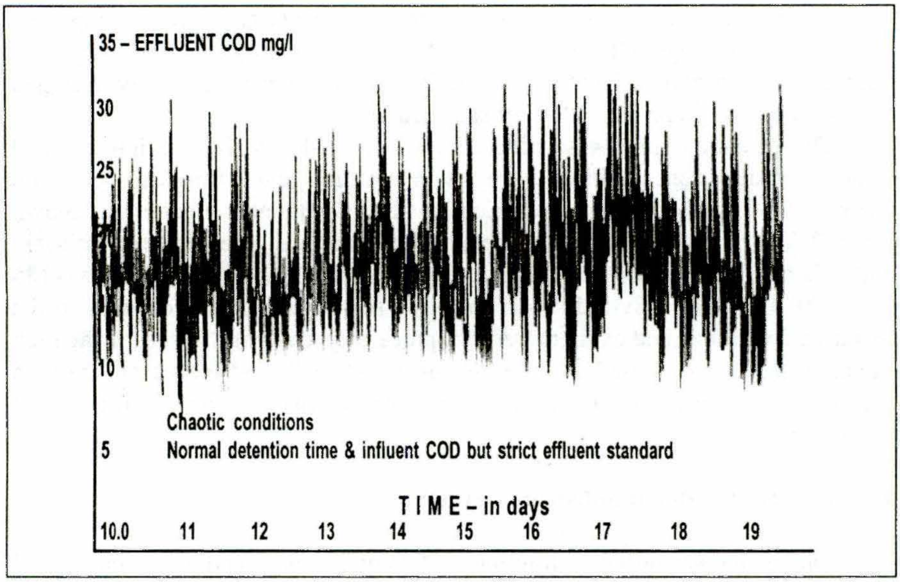

Figure 8. Effluent COD concentration Fluctuation with Stringent Effluent Quality Standard

High concentration of microbial mass is required in the aeration tank to remove the COD to very low concentration. However, the rate of microbial decay also increases that there are instances wherein the recycled sludge are mostly inert solids. In practice the effluent quality from the activated sludge is restricted to 20 mg/l although some extended aeration system operating at long detention time are capable of producing effluents with COD concentration of 5 to 10 mg/l. Designing activated sludge systems to produce effluent COD concentration lower than 5 mg/l to 10 mg/l. Designing activated sludge systems to produce effluent with COD concentration of 5 to 10 mg/l. Designing activated sludge systems to produce effluent COD concentration lower than 5 mg/l are highly unreliable.

C. Effects of Influent Substrate Concentration

The model was run with the influent COD concentration increased to 750 mg/l while the detention time was maintained at 0.25 day and the target effluent COD concentration at 20 mg/l. The results of the model are shown in Fig. 9. The model showed chaotic behavior under this condition. The effluent COD concentration fluctuated erraticially. The behavior of the model explains the need for a pre treatment of strong wastes specially industrial wastes prior to treatment in the activated sludge process. This observations justifies designers preference to use the extended aeration variation of the activated sludge in treating wastewater with high concentration of organic matter. The extended aeration process has

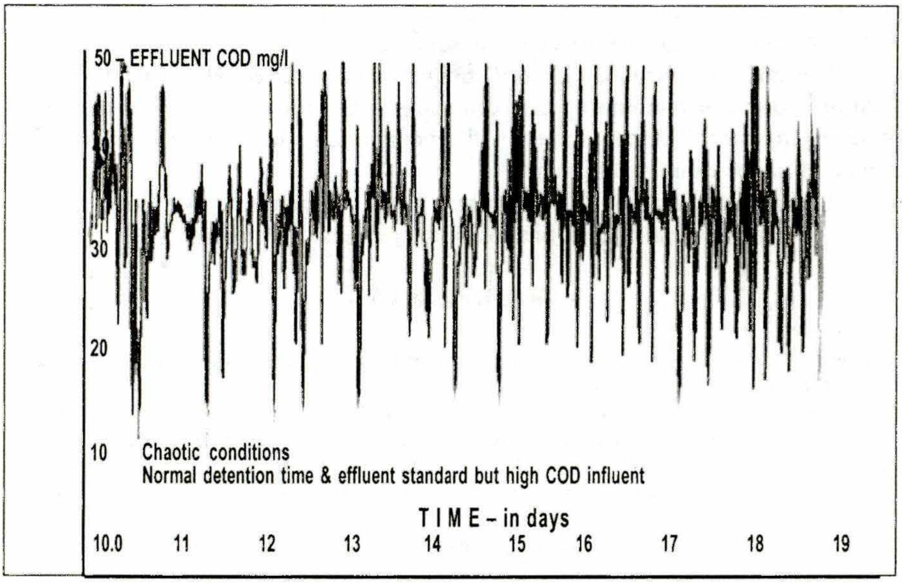

Figure 9. Effluent COD concentration Fluctuation at High Influent COD Concentration

detention time from 0.75 to 2 days compared to the simple completely mixed activated sludge system with detention times of 0.16 to 0.5 day. The completely mixed activated sludge system when treating wastewater with high concentration of organic wastes tends to show erratic behavior.

CONCLUSION

At low detention times similar to those at lower range of the high rate activated sludge process exhibits chaotic behavior. Activated sludge process treating 250 mg/l of influent COD as in typical domestic sewage could still be used as a pre treatment unit.

As the activated sludge system exhibits chaotic behavior the development of complicated models involving more non-linear differential equations should be evaluated properly. The utility of those models when the system exhibits chaotic behavior is minimal.

RECOMMENDATION FOR FURTHER STUDIES

The fluctuations in the influent flow rate and substrate quality increase the non-linearity of the substrate and active microbial mass population. The fluctuations in the composition of the recycled active microbial mass concentration and the influent flow rate and substrate quality could have a resonance effect creating pockets of chaotic behavior within a stable region. The effect of those fluctuations on the activated sludge process could be studied.

The effect of introducing more complex non-linear equations in the model such prey predator relationship between bacteria and protozoa, and the oxidation of nitrogen to nitrite and nitrate on the boundaries where chaotic behavior is exhibited can be examined further.

The model bifurcation diagram and Poincare maps may be developed to improve the mapping and identification of the regions of chaos.

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